

## Range Image Registration by using an Edge-Based Representation

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**Abstract.** This paper proposes a new strategy to register range images by using an efficient edge-based representation. It consists of two stages. First, a fast edge-based segmentation technique extracts the crease and jump edge points from the surfaces contained in the given range image. The segmentation technique is based on a scan line approximation algorithm. It generates a binary edge map. The second stage computes the registration parameters by using the classical ICP algorithm but taking into account only the discontinuity points. There is a considerable difference with the previous approaches, as only the points defining edges are considered instead of all the range image points. That difference is shown in the CPU time required to compute the registration parameters and, moreover, in the sensibility to the initial estimate of the transform between the range images to be registered. Experimental results with different real range images are presented. Moreover, a brief comparison between edge-based registration versus cloud of points registration is given.

### 1 Introduction

In order to capture the full geometry of a complicated object and then generate a 3D model of it, several range images may be required. Prior to merging that information into a single 3D model, it is necessary to apply a transformation to the data acquired from different views in order to express them into the same reference frame. This process, known in the literature as the registration task, involves finding the six parameters (three rotations and three translations) which properly transform the local reference frames associated with each range image to another reference frame associated with the reconstructed model (generally, the first sensor position).

Assuming that there is enough overlapping area between the range images, several registration approaches have been proposed in the literature during the last decade. They can be classified into two different groups: coarse registration and fine registration techniques. *Coarse registration techniques* are based on finding a match between distinguishing features that may be present in the different views. These techniques do not require any prior knowledge of the transformation. *Fine registration techniques* include those approaches that consider the registration task as an optimization problem. These approaches do not require to extract features from the range images, but, they are based on the assumption that a transformation between two views is known beforehand, or can be estimated. Thus, a cost function is defined, which measures the quality of the alignment between the partial overlapped surfaces contained in each view. The range images are then registered by determining the 3D rigid transformation which minimizes this cost function. The differences between the various proposals are based on the definition of the cost function.

The best-known fine registration technique is the *iterative closest point* algorithm (ICP). It has been

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originally presented by [1]; since then, several variations based on other cost functions have been presented (e.g., [2], [3], [4], [5], [6], [7]). For example [6] proposes to minimize a function which is defined by both the 3D distance between the points that have been matched and the difference between the maximal principal curvature associated with each one of those matched points. A more general approach has been proposed by [7]. There ICP algorithm is carried out considering a  $\mathcal{R}^{3+n}$  space, where  $n$  is the number of invariant attributes that describe each point. In that work moment invariants were used. Although experimental results have shown that this algorithm converges in fewer iterations than the classical ICP, there is an additional overhead involved in working in the higher dimensional space ( $3+n$  space) thus on an average, the CPU time is equal or worse than ICP algorithm in 3D space.

ICP algorithm has an average complexity of  $O(n \log n)$  for  $n$  point images. Hence, in order to obtain a more efficient technique, the current work proposes to use ICP algorithm considering only a subset of points, the more pertinent ones—a difference with the previous approaches where the whole points are considered. This subset is defined by the edge points—points where depth or orientation discontinuities appear. The proposed technique first extracts the edges from the two range images to be registered. It is performed by using a fast edge-based segmentation technique. Then, only over those extracted points the ICP algorithm is computed. This strategy is advantageous not only in terms of processing speed-up, but also in terms of the sensibility to the initial estimated transformation used to start ICP algorithm. In some cases it has been shown that when this initial estimate is very far from the correct value, the ICP algorithm, using the whole points, cannot succeed in converging on the registration parameters [8]—it is trapped by a local minimum. However, in those situations, the correct parameters can often be found by ICP when only the edge points are used.

This work is organized as follows. Section 2 presents the scan line approximation technique applied to compute the edge-based representation. Section 3 gives a brief summary of the ICP algorithm used to compute the registration parameters. Section 4 shows experimental results by using different range images obtained from Stanford 3D scanning repository and OSU (MSU/WSU) database. Moreover a brief comparative study of range image registration by using the original cloud of points versus the proposed technique is presented. Finally, conclusions and further improvements are given in section 5.

## 2 Edge-Based Representation

An edge-based representation provides a compact and rich information about the surfaces contained in a given range image. Such representations have been used in applications such as object recognition [9], 3D modeling [10] or to guide and improve surface based segmentation (e.g. [11], [12]) to mention a few. In this sense the current work proposes the use of an edge-based representation to tackle the range image registration problem.

This section describes the technique used for obtaining the edge-based representation. It consists in generating a two dimensional array  $\mathbf{B}$ , where each element is a binary value indicating whether that point is an edge point or not. This stage has been implemented by using a scan line technique similar to the one presented in [13][14]. A scan line is a profile digitalization where its 2D projection can be represented by a planar curve. By analyzing that curve it is possible to detect *crease* and *jump* edge points. Crease edges are those points in which a discontinuity in the surface orientation appears while jump edges are defined by a discontinuity on the surface. Previous works dealing with scan line approximation were presented in [11][13][15]. In [15], each scan line is approximated by means of a set of straight line segments; only the points contained in the rows are used to define each scan line. On the contrary [11] and [13] approximate the scan line by using quadratic functions. In both works, rows, columns and diagonals are considered as scan lines. In the current work, as the same as in [14], only rows and columns are considered as scan line—but not diagonal. Every edge contained in the image can be obtained by analyzing only two orthogonal directions.

At this stage, every row and column (hereinafter called scan line) is approximated by a set of oriented quadratic functions. Quadratic functions have been selected due to they allow to generate a more generic edge based representation than if only straight line segments were considered; moreover for range images acquired from an industrial site (especially, a chemical factory), second-order surfaces are

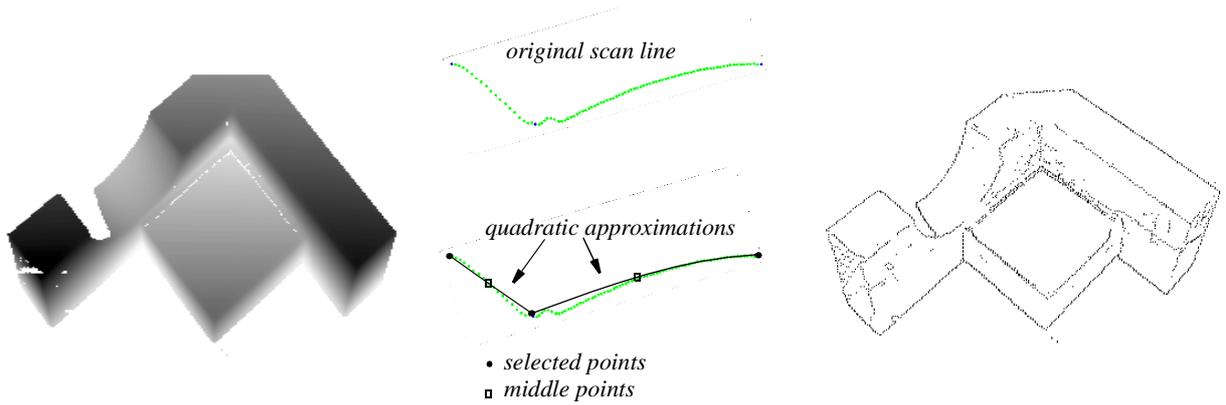


Figure 1. (left) Original range image (rendered). (middle) Illustration of a scan line approximated by two quadratic expressions. (right) Edge-based representation.

numerous and quadratic approximations of the edges allow to reduce the number of points to be considered during the registration stage. From those curves the representative points are extracted and marked in the binary edge map. The algorithm consists of two steps. First, the jump edge points are detected using a threshold adapted with respect to the depth of the points; these points are used to cut the original scan line into a set of sections (set of consecutive points) and to define the starting and ending points of each one of them. Second, a recursive algorithm approximates each section (set of points), by quadratic functions oriented along edge  $y$ :

$$y = ax^2 + bx + c \quad (1)$$

This algorithm is applied to every section separately. It has been implemented by means of the classical recursive splitting algorithm [17]. A quadratic function is obtained by using the first, middle and last points of the considered scan line' section. Let us note  $(P_{f(x,y)}, P_{m(x,y)}, P_{l(x,y)})$  as the first, middle and last points. Then, before obtaining the parameters of function (1), the set of points contained into that limits are rotated around the first point until the configuration:

$$P_{f(y)} = P_{l(y)} > P_{m(y)} \quad (2)$$

is reached (see illustration in Fig. 1(middle)).

Next, the parameters of function (1) are obtained analytically by using those three points. The approximation error between the obtained quadratic function and every rotated point is computed. If this error is greater than a given threshold  $\xi$ , the set of points is split into two set of points at that position where the biggest error appears. Then, that position is now considered as last point, the middle point is computed, the points between the first and the new last points are rotated according to (2), and the parameters of (1) are computed again. This splitting algorithm is applied recursively while the approximation error is greater than  $\xi$ .

The result of this recursive algorithm is a set of quadratic curves approximating the considered scan line's section. Once this section is approximated, the recursive algorithm is carried out over the next section of the given scan line. From each quadratic curve, the first and last points—points used to compute the parameters of function (1)—are selected and their positions in a binary map  $\mathbf{B}$  are marked. The original range image can be provided by any 3D sensor, like stereovision or laser range finder. In this way, the binary edge map can be represented as a two dimensional array  $\mathbf{B}$ , where each element  $\mathbf{B}(r,c)$  is a binary value indicating whether the point has been selected by the previous approximation technique or not.

Once a given scan line has been approximated, the algorithm starts again over the next new scan line. When all the scan lines—rows and columns—have been processed, the obtained binary edge map is the final result and the next stage is applied. Fig. 1(right) shows the binary edge map—edge-based representation—obtained after processing the range image shown in Fig. 1(left). This image has been acquired on a polyhedral object; points close to the object edges have been marked, but due to the sensor noise, some other points on the faces have also been detected as discontinuity points

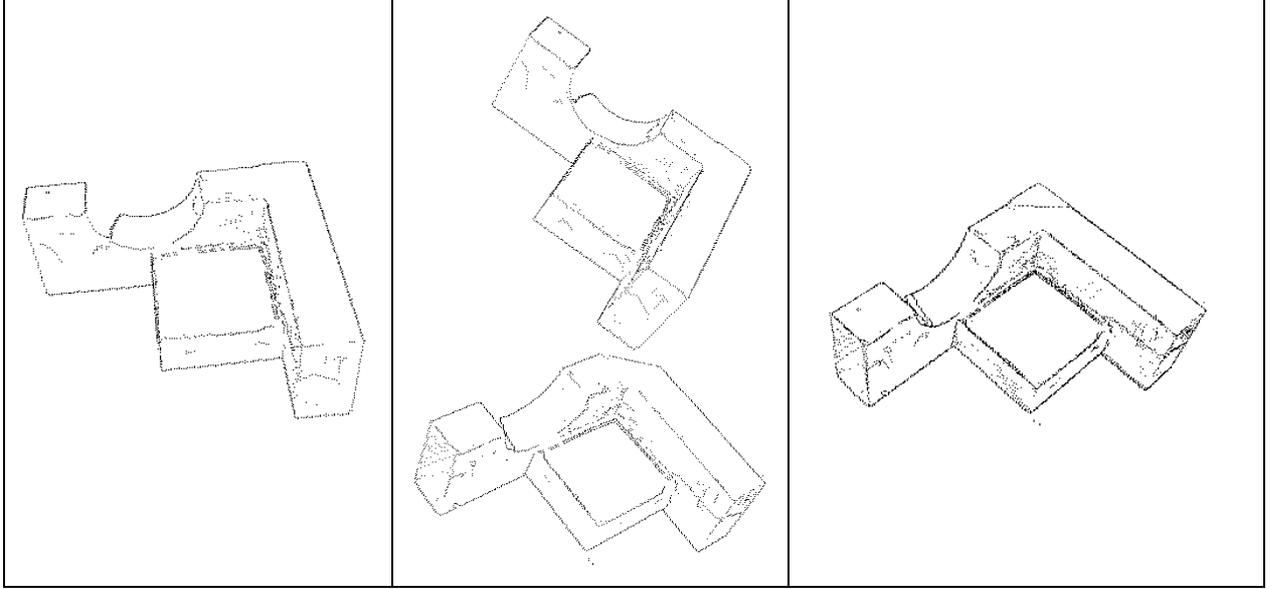


Figure 2. (left) Edge-based representation from a range image defined by 204x232 points. (middle) Original position of the two range images—Fig. 1(right) and Fig. 2(left)—used as starting point by ICP algorithm. (right) Range images after the registration process.

### 3 Edge Registration

Let  $B_i(r;c)$  and  $B_j(r;c)$  two binary edge map representations corresponding to two different viewpoints,  $i$  and  $j$  respectively, the first one referred to a global coordinate frame ( $i$ ) and the second one referred to its local coordinate frame ( $j$ ). The objective at this stage is to obtain the parameters of a matrix  $\mathbf{T}$ —expressing a rotation  $\theta$ , and a translation  $\Gamma$ —which allow to express the points ( $P_j$ ) acquired from viewpoint  $j$  in the reference frame  $i$ . In order to obtain those parameters, the ICP algorithm has been implemented. In [1], an exhaustive study of point set matching, curve matching and surface matching by using ICP has been presented. There, it has been proven the monotonic convergence for all those cases. By using that results, in the current implementation we focus in the curve matching but by using only those curves defining edges.

The parameters of transformation matrix  $\mathbf{T}$  are computed by means of the following iterative process. It is applied while the *registration error* is higher than a given threshold. First, an initial transformation matrix  $\mathbf{T}_0$ —with parameters  $\theta_0$  and  $\Gamma_0$ —is estimated. With this matrix, the points from viewpoint  $j$  are transformed and for each point  $P_j$  the closest point  $P_i$  is identified. The closest point detection has been implemented by using the statistical filtering technique proposed by [2]. Next, couples of matched points ( $P_i, P_j$ ) are used to compute the registration error:

$$\zeta_{\mathfrak{R}} = \sum_n (P_i, P_j) \quad (3)$$

where  $n$  is the number of matched points. If  $\zeta_{\mathfrak{R}}$  is below or equal to the given threshold, matrix  $\mathbf{T}_0$  is the final solution and parameters ( $\theta_0, \Gamma_0$ ) are used to express the points  $P_j$  in the reference frame  $i$ . Otherwise, if  $\zeta_{\mathfrak{R}}$  is higher than the given threshold, couples ( $P_i, P_j$ ) are used to compute a new set of parameters ( $\theta_j, \Gamma_j$ ) by minimizing the next expression [16], using a linear method proposed in [18]:

$$\Sigma^2 = \sum_n \|P_i - (\theta_j P_j + \Gamma_j)\|^2 \quad (4)$$

again,  $n$  represents the number of matched points. With this new set of parameters ( $\theta_j, \Gamma_j$ ) a new transformation matrix  $\mathbf{T}_j$  is computed, and the process starts again from the beginning (now considering the

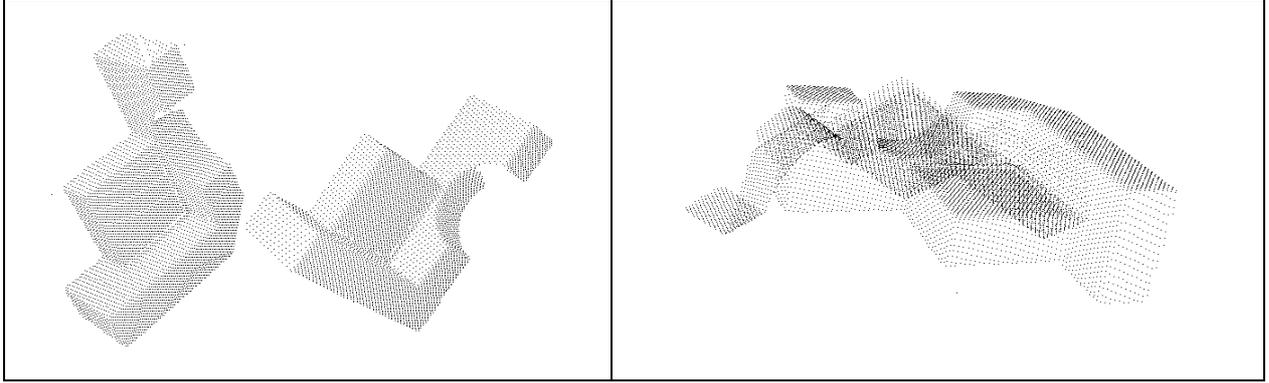


Figure 3. (*left*) Original position of the range images to be registered (all the points were considered). (*right*) Final position computed by using the registration parameters obtained with ICP

obtained matrix  $\mathbf{T}_l$ ). This iterative process is executed until the convergence on a  $\mathbf{T}_l$  matrix for which the error is lower than the given threshold, or until a maximum number of iterations is reached (non-convergence).

## 4 Experimental Results

The proposed technique has been tested with real range images obtained from different databases (OSU (MSU/WSU) and Stanford 3D scanning). CPU times have been measured on a Sun Ultra 5. Fig. 1(*right*) shows the edge-based representation for the range image shown in Fig. 1(*left*). That range image is defined by 225x216 (rows x columns) points. The edge-based representation was obtained by using the technique presented in Section 2, it contains 1,751 points; the CPU time necessary to extract those points was 1.41 sec. Fig. 2(*left*) shows another edge representation for the same object but extracted from a different point of view, the original range image is defined by 204x232 points. That representation was computed in 1.03 sec. and contains 1,584 points. Next, both edge-based representations—Fig. 1(*right*) and Fig. 2(*left*)—were registered by ICP algorithm considering as initial estimated position that showed in Fig. 2(*middle*). The registration parameters were computed in 54.03 sec. These parameters were used to put the edge points extracted from the second range image in the reference frame of Fig. 1(*right*), the obtained result is shown in Fig. 2(*right*). The registration errors (rotation and translation) obtained with the proposed technique can not be computed with respect to a ground truth, due to the fact that no information about the real registration parameters is given in the OSU database. By using the initial position showed in Fig. 2(*middle*) the ICP algorithm was run but now considering all the points defining each range image (see Fig. 3(*left*)). In this last case, ICP can not compute the correct registration parameters because that original position, which is used as initial estimated transformation between the two views, is too far from the final one. The classical ICP has been trapped in a local minimum (the result obtained after 354.57 sec. is shown in Fig. 3).

Fig. 4(*left*) shows initial positions of two edge-based representation (816 and 1,206 points) which were registered by using the proposed technique. They were obtained in 0.54 sec. and 0.98 sec. respectively and correspond to range images defined by 162x222 and 211x232 points. The difference with the previous example, is that the overlapping area between the surfaces in each viewpoint is smaller. However, in spite of that, the registration by using the edge representation has converged on the correct parameters. The registration parameters were computed by ICP algorithm in 29 sec., with these parameters the edge points from the second range image were transformed and the result is presented in Fig. 4(*middle*). Fig. 4(*right*) shows the reference frame error (translational error) versus CPU time for registering the range images showed in Fig. 4(*left*) considering only the edge points in one case and all the range image points in the second case. The reference frame error was computed considering a prior knowledge of the final reference frame position. Then, at each iteration the difference between the current reference frame position and the final one was used to define the frame error at that stage. Obviously,

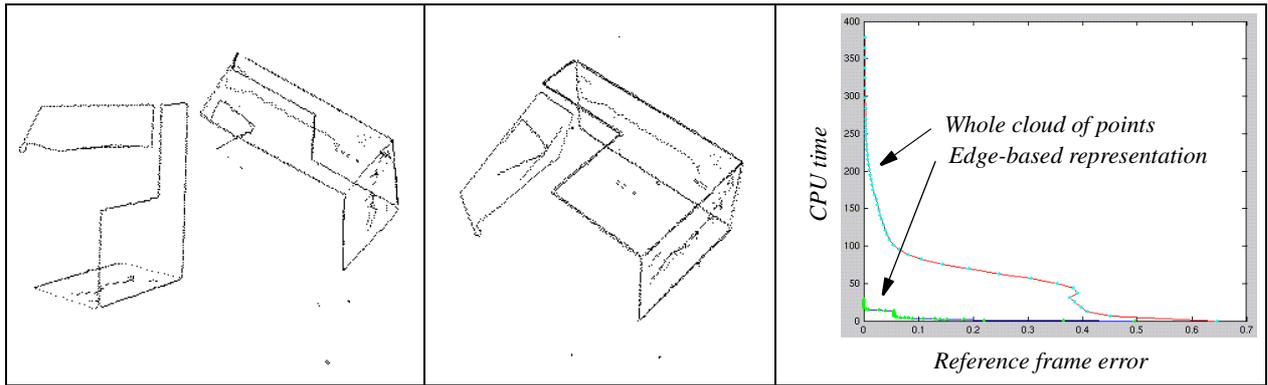


Figure 4. (left) Original position of two edge-based representations which will be registered. (middle) Result obtained after the registration process. (right) Reference frame error versus CPU time for registering range images shown in Fig. 4(left) by using only the edge points or all the cloud of points (in both cases the original position of the pair of range images has been the same).

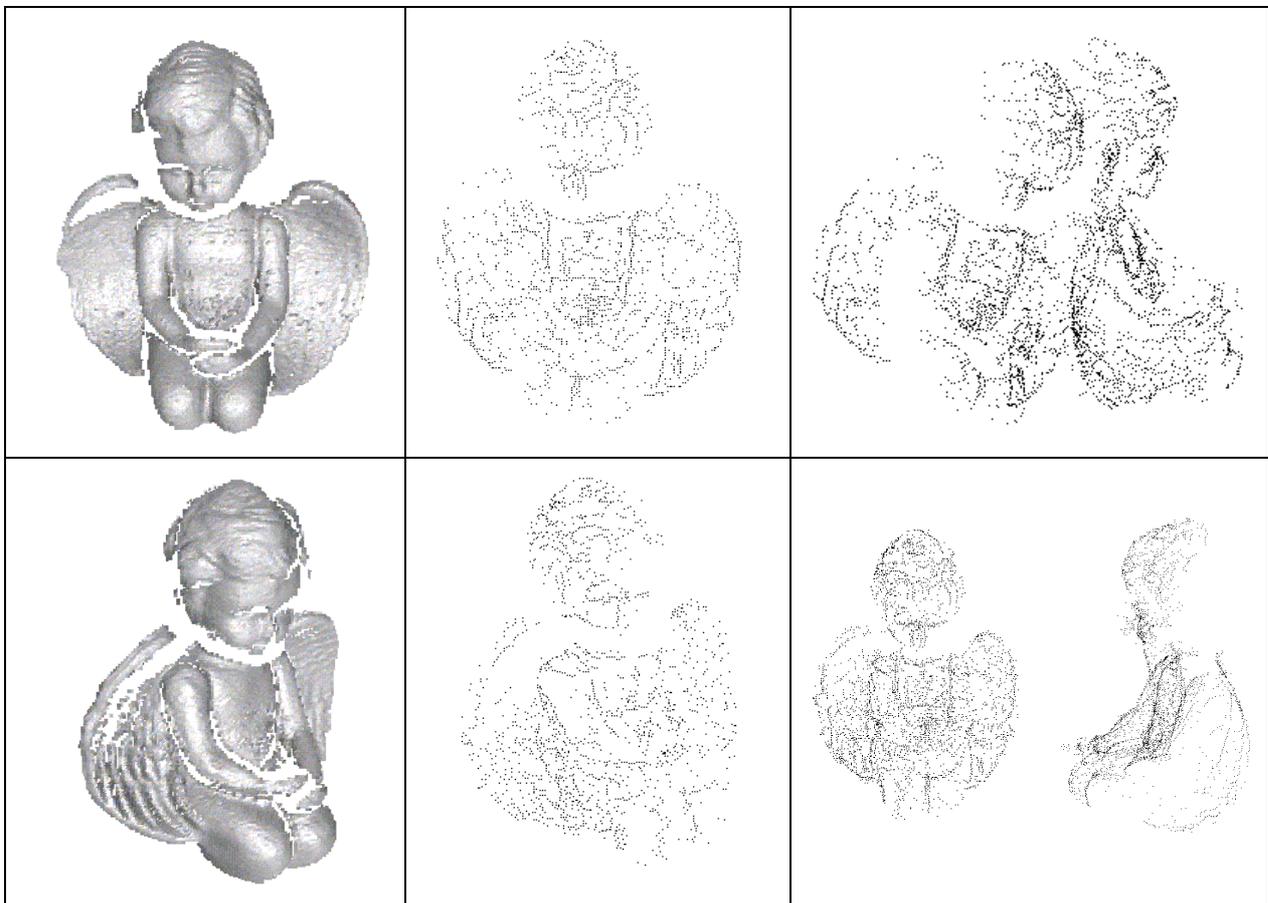


Figure 5. (left) Original range images taken from two different viewpoints. (middle) Edge based representations (only crease edge points are considered). (top-right) Initial position for ICP algorithm. (bottom-right) Final result obtained by using the parameter registration computed by ICP (two views).

the convergence is reached 5 times faster using only the edge points.

Fig. 5 and Fig. 6 give examples by using sculptured objects obtained from OSU (MSU/WSU) database. These examples have been used to test the proposed technique when free-form objects are considered. In both cases, the registration errors obtained with the proposed technique could not be computed due to the fact that the real registration errors are unknown. The difference with the previous

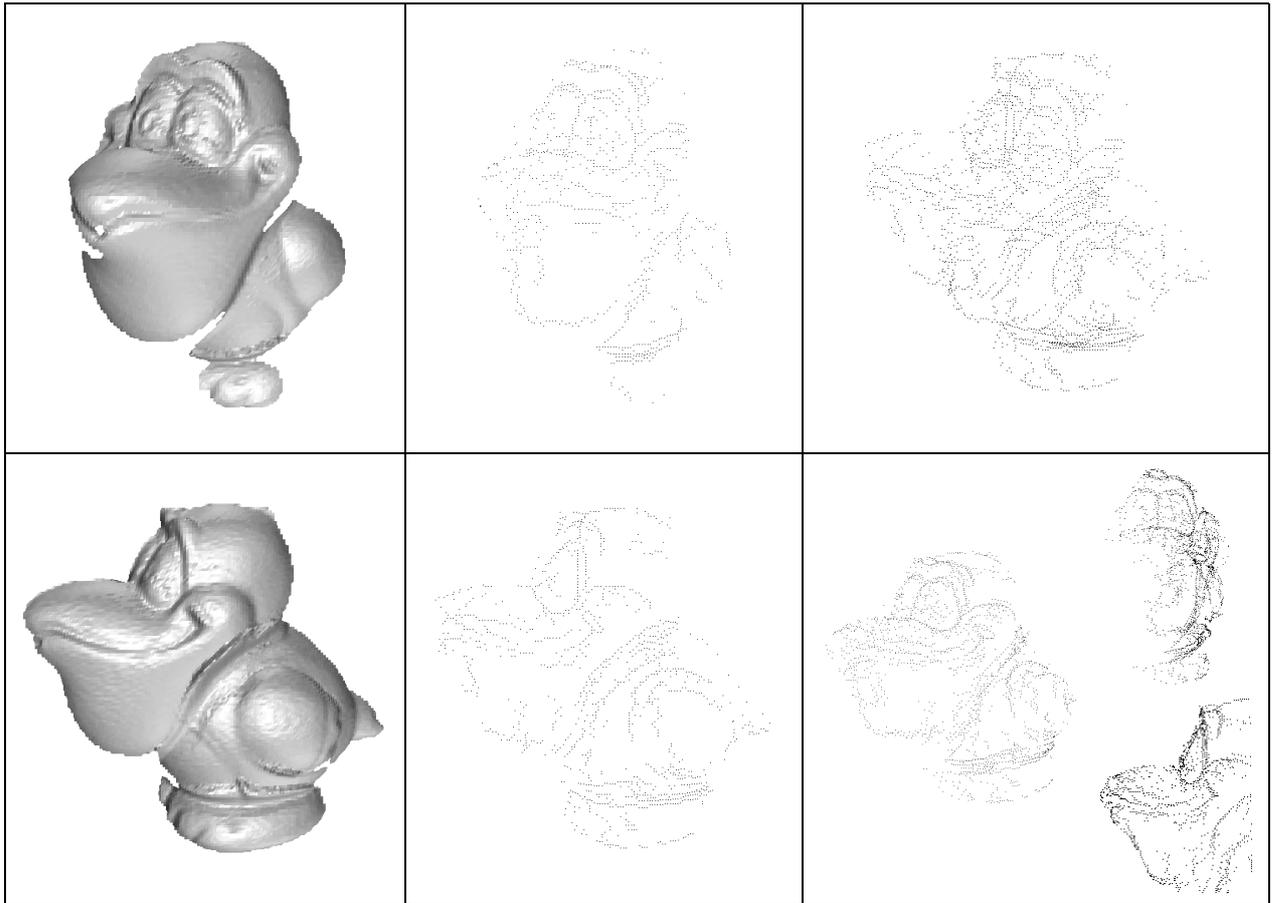


Figure 6. (*left*) Range images taken from two different viewpoints. (*middle*) Edge based representations (only crease edge points are considered). (*top-right*) Initial position for ICP algorithm. (*bottom-right*) Final result obtained by using the parameter registration computed by ICP (3 views).

examples is that the binary edge representations are obtained by considering only the crease edges (first and last points from each quadratic curve; the jump edge points are discarded). The latter, was due to the fact that in range images containing free-form objects the jump edges can correspond to occlusion lines; therefore, points belonging to jump edges could produce a false matching leading to a bad registration. The obtained results prove that the presented technique also can deal with this kind of objects. Fig. 5(*left*) shows two range images defined by 200x200 points each (rendered). Fig. 5(*middle*) shows the edge-based representations corresponding to the previous range images. They are defined by 1,746 and 1,634 points respectively (without jump edges). The edge-based representation stage took 1.22 sec. for the first image and 1.46 sec. for the second one. Fig. 5(*top-right*) shows the initial position considered as starting point by ICP algorithm—initial estimated transformation. The registration parameters were used to put in the same reference frame the data points of Fig. 5(*top-middle*) and (*bottom-middle*). They were computed in 50.42 sec. The resulting image is shown in Fig. 5(*bottom-right*). Fig. 6(*left*) shows other two range images (rendered) which were registered by using the proposed technique (200x200 points each). The edge based representations were obtained in 0.13 sec. and 0.22 sec. respectively. They are defined by 894 and 1,191 points (see Fig. 6(*middle*)). The registration parameters were computed by ICP in 48.02 sec. Fig. 6(*top-right*) shows the initial position used as estimated transformation by ICP, the final position is shown in Fig. 6(*bottom-right*).

Finally, Fig. 7 shows the results obtained by processing different range images obtained from Stanford 3D scanning repository. These range images are defined by 400x512 points each (204,800 points). They correspond to a sequence of four images generated by scanning a single object from four different point of views. Fig. 7(*left*) shows the edge-based representations obtained by only considering the crease edges. Notice that this particular object is very rich in crease edges, but in spite of that the points which

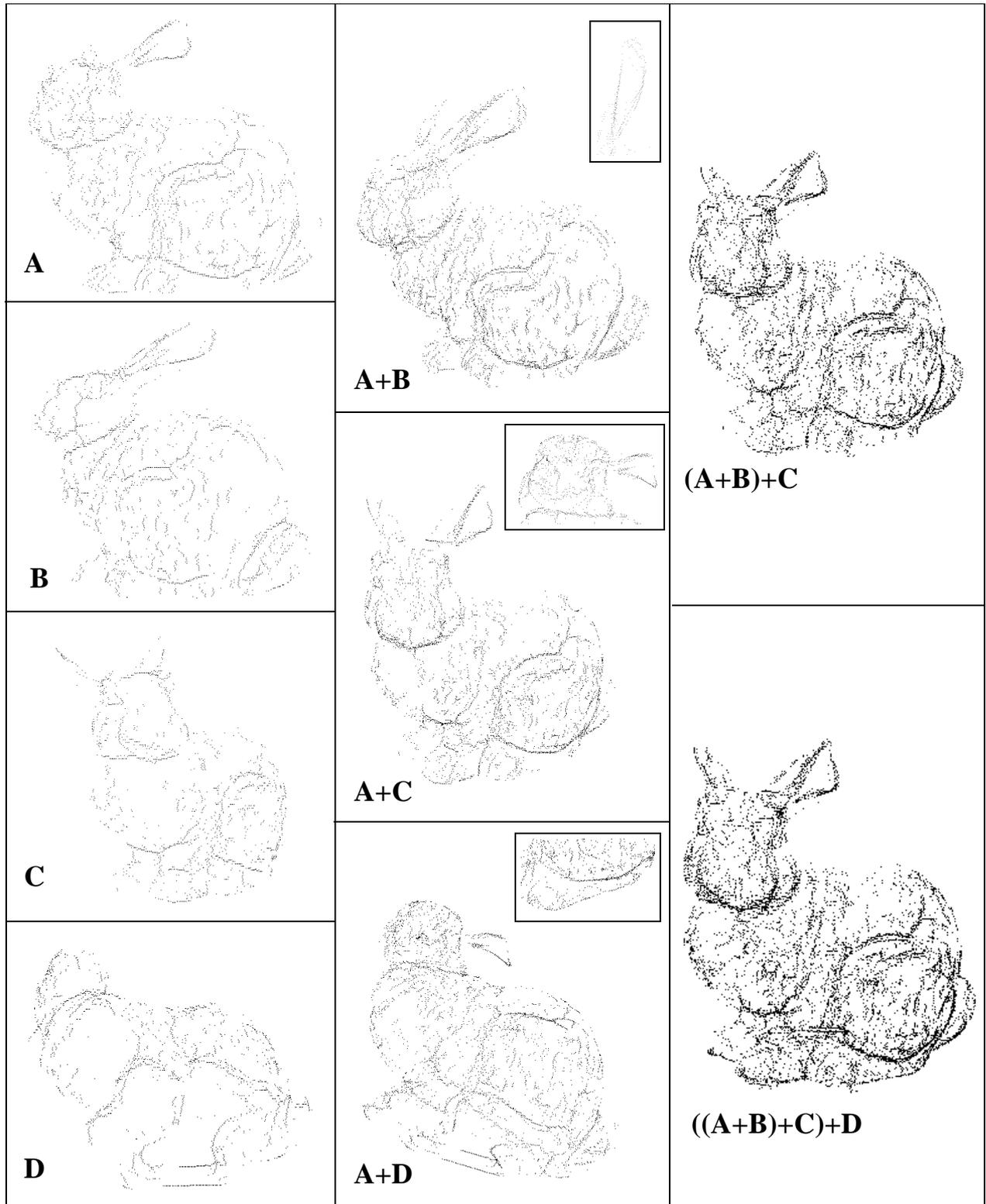


Figure 7. (*left*) Sequence of edge based representations corresponding to a single object scanned from four different views: A, B, C and D. (*middle*) Results from the pair-registration of the left views; some details are showed for each result. (*right*) Results from an incremental registration.

have to be considered by ICP are less than 1.12% of the total number of points. These edge-based representations are defined by 2,295 points, 2,010 points, 1,588 points and 1,737 points respectively. The CPU time to obtain them was 1.15 sec., 1.11 sec., 0.88 sec. and 0.93 sec. The reference frame associated with

Fig. 7(*top-left*) has been used as the global reference frame by ICP algorithm. Thus, the other three images showed below are registered with this first one pairwise, the results of the registration are showed in Fig. 7(*middle*). Some details have been enlarged to focus on difficult areas. Fig. 7(*right*) shows the final result of the incremental registration of the views on the left: the result of the registration of the first two upper left views (A+B) is registered with the third ((A+B)+C) and this result to the fourth (((A+B)+C)+D).

Table 1. shows all the information relative to the registration of the previous views: CPU time, number of iterations, rotational and translational errors; as it was mentioned before the errors are relative to the values given in the Stanford repository.

Registration				Results			CPU			Errors	
Image1	Pts	Image2	Pts	Image	Pts	%Pts match	iter ICP	Tot [sec]	T/iter [sec]	Rot rms	Trans rms
A	2295	B	2010	A+B	4305	38.26	25	42.50	1.70	9.8538E-04	1.3014E-03
A	2295	C	1588	A+C	3883	32.68	70	93.46	1.34	1.2302E-04	2.9405E-04
A	2295	D	1737	A+D	4083	23.66	60	105.30	1.76	8.7469E-03	1.1581E-03
A+B	4305	C	1588	(A+B)+C	5893	20.28	70	189.44	2.71	3.0323E-03	2.8637E-04
A+B	4305	D	1737	(not shown)	6042	15.15	60	205.66	3.43	3.3690E-03	8.9983E-04
(A+B)+C	5893	D	1737	((A+B)+C)+D	7630	15.07	60	285.77	4.76	4.5458E-03	8.8853E-04

Table 1. Registration results of the views showed in Fig 7. The first three lines correspond to the registration between A and (B, C, D); the following three lines show the results of the incremental registrations. The number of points from each image, the final percentage of matched points, CPU time and rotational and translational errors in reference to the given transformation are presented.

## 5 Conclusions and Further Improvements

This paper presents a new strategy to register range images by using both, an edge-based representation and the classical ICP algorithm. The proposed technique has been tested with different kind of range images showing a good performance in front of the classical registration techniques which use all the range image data points. The advantages of the proposed technique are in terms of CPU time as well as in terms of the sensibility to the initial estimated transformation. It has been shown that ICP algorithm could not always converge to the correct solution when all the points contained in the given range image are considered; however in these particular situations ICP found the correct parameters when an edge-based representation was considered (in both cases the same  $T_0$  matrix has been used).

Edge based representation is a compact way to describe the different views of a single object or a scene. However, due to variations in surface sampling and noise from the range sensor, these edge based representations will not be exactly the same. Then, small registration error will appear. In this sense, a further work will consist of merging the proposed approach with the classical registration technique (considering all the range image points). The objective is to obtain in a first attempt, and in a fast way, a good registration transformation by using the proposed technique. Next, if a better accuracy is required, when the range images to be registered are almost near to the final position all the data points will be considered improving thus the final result—alternatively points to be registered could be included in an incremental way according with some weight function.

Another further work will be to study those cases where the density of points in the range images to be registered is different. For those particular cases an edge resampling strategy will be implemented. It will generate the same density of points in both edge representations to be registered. This process could be understood as a decimation algorithm used for the techniques which register range images represented by triangular meshes [18].

## 6 References

- [1] P. Besl and N. McKay, A Method for Registration of 3-D Shapes, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 14, no. 2, pp. 239-256, February 1992.
- [2] Z. Zhang, Iterative Point Matching for Registration of Free-form Curves and Surfaces, *Int. Jour. Computer Vision*, vol. 13, no. 2, pp. 119-152, 1994.
- [3] R. Bergevin, M. Soucy, H. Gagnon and D. Laurendeau, Towards a General Multi-View Registration Technique, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 18, no. 5, pp. 540-547, May 1996.
- [4] C. Dorai, J. Weng and A. Jain, Optimal Registration of Multiple Range Views, *Int. Conf. on Pattern Recognition*, 1994.
- [5] O. Carmichael and M. Hebert, Unconstrained Registration of Large 3D Point Sets for Complex Model Building, *IEEE Int. Conf. on Intelligent Robots and Systems*, Victoria, Canada, October 1998.
- [6] B. Sharp, S. Lee and D. Wehe, Invariant Features and the Registration of Rigid Bodies, *IEEE Int. Conf. on Robotics and Automation*, Detroit, 1999.
- [7] R. Yang and P. Allen, Registering, Integrating, and Building CAD Models from Range Data, *IEEE Int. Conf. on Robotics and Automation*, Leuven, Belgium, 1998.
- [8] J. Bozier, M. Devy and A. Sappa, A Geometrical Approach for the Incremental Modelling of Free Form Surface by Triangular Meshes, *Int. Symp. on Intelligent Robotic Systems*, Reading, UK, July 2000.
- [9] S. Pankanti, C. Dorai and A. Jain, Robust Feature Detection for 3D Object Recognition and Matching, *Proc. of SPIE Conference on Geometric Methods in Computer Vision*, San Diego, pp. 366-377, July 1993.
- [10] A. Hoover, D. Goldgof and K. Bowyer, Extracting a Valid Boundary Representation from a Segmented Range Image, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 17, no. 9, pp. 920-924, September 1995.
- [11] O. Pereira Bellon, A. Direne and L. Silva, Edge Detection to Guide Range Image Segmentation by Clustering Techniques, *IEEE Int. Conf. on Image Processing*, Kobe, Japan, October 1999.
- [12] A. Sappa, V. Bevilacqua and M. Devy, Improving a Genetic Algorithm Segmentation by means of a Fast Edge Detection Technique, *IEEE Int. Conf. on Image Processing*, Thessaloniki, Greece, October 2001.
- [13] X. Jiang and H. Bunke, Edge Detection in Range Images Based on Scan Line Approximation, *Computer Vision and Image Understanding*, vol. 73, no. 2, pp. 183-199, February 1999.
- [14] A. Sappa and M. Devy, Fast Range Image Segmentation by and Edge Detection Strategy, *Third Int. Conf. on 3D Digital Imaging and Modeling*, Québec, Canada, May 2001.
- [15] X. Jiang and H. Bunke, Fast Segmentation of Range Images into Planar Regions by Scan Line Grouping, *Machine Vision and Applications*, vol. 7, no. 2, pp. 115-122, 1994.
- [16] K. Arun, T. Huang and S. Blostein, Least-Squares Fitting of Two 3-D Points Sets, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 9, no. 5, pp. 698-700, September 1987.
- [17] D. Duda and P. Hart, *Pattern Classification and Scene Analysis*, Wiley, New York, 1972.
- [18] A. Johnson and M. Hebert, Surface Registration by Matching Oriented Points, *Int. Conf. on Recent Advances in 3-D Digital Imaging and Modeling*, Ottawa, Ontario, May 1997.
- [19] R.M. Haralick, H.Joo, C.Lee, X.Zhuang, V.G. Vaidya and M.B. Kim, Pose Estimation from Corresponding Point Data, *IEEE Transactions on Systems, Man and Cybernetics*, Vol.19, No., p.1426-1445, nov.1989